

Past, Present and Future

On the occasion of ESSLLI's 25th Birthday,
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Arthur Prior



Today we'll explore the work of Arthur Prior in tense logic and hybrid logic — but the point is not to enumerate or appraise his work, rather it is to ask: *What was logic to Prior? How did his conception differ the ESSLLI conception? And what will the logicians be talking about at ESSLLI 2038?*

Long ago and far away...



Prior was born in Masterton, New Zealand — even today an archetypal example of a New Zealand country town. Located in the Wairarapa region, it's cut off from the capital by the Rimutaka ranges. The word 'isolated' works well here...

Arthur Prior

Born 4 December, 1914, Masterton, New Zealand.
1949 Work veers towards philosophy and logic.
1953/54 Developed first versions of tense logic.
1956 John Locke Lectures, Oxford.
1957 Publication of *"Time and Modality"*.
1967 Publication of *"Past, Present and Future"*.
1968 Publication of *"Papers on Time and Tense"*.
Died 6 October, 1969, Trondheim, Norway.

What is Tense Logic?

A 'modal style' logic of time:

$F\phi$ means at sometime in the future.

$P\phi$ means at some time in the past.

$G\phi$ means all times in the future.

$H\phi$ means all times in the past.

Semantics? Ah...

Little creatures in time...

An **internal** perspective on time.

We stand **inside** time, at the **point of speech** or the **deictic centre**. And the **points of event** is located relative to this.

It's an **A-series** view of time, a **we-live-here-in-the-flow**, view of time. And this A-series view is what his tense logic reflects. It's not a **God's-eye-view**, or **B-series** perspective: it's located, situated.

Nowadays, many modal logicians claim that this internal perspective is right at the heart of modal logic: that this is what **Kripke semantics**, and **van Benthem's work on bisimulation** tells us. But all that came much, much later. Prior's semantic insight helped write the blueprint for these developments.

And Prior played, and learned...

That $FF\phi \rightarrow F\phi$ 'means' **time is transitive**, that $F\phi \rightarrow FF\phi$ 'means' **time is dense**. But nothing seems to 'mean' that time is irreflexive.

That thinking about time as branching towards the future in various ways enabled us to avoid the spectre of determinism (a psecter that haunted Prior).

That tense logic could be extended to logics that measured time.

That indexicality truly is essential.

Quite simply, "*Past, Present and Future*", Prior's most cited book, is a feast. Written by a philosopher-logician at the height of his powers, and giddy with love for his subject.

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The A series vindicated!

So there we have it. **The language of time!** And it's an A-series language! And it works! And it's beautiful and surprising.

And Prior did view his language as the language of time, in much the same way as a Martin Löf type theorist might view type theory as the language of mathematics. Model theoretic semantics — translations into ZFC! — aren't the point. **The languages are foundational.**

So all is well in the garden...?

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Here's the story...

- A series talk (Prior's beloved tense logic) is **weaker** than B series talk.
- This is a problem for Prior. Not a big one, but a problem.
- Arthur Prior invented **hybrid logic** to solve this problem. Which it did. Beautifully.
- He then discovered that hybrid logic had given him an (deeper) problem. **Prior's nightmare.**
- Nothing in his published writing fully solves the new difficulty. Perhaps he would have found answers had he lived to complete "Worlds, Times and Selves".
- Hybrid logic was **not** an optional extra for Prior: appreciating its role is crucial to understanding his work. **And he found its role problematic.**

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What is B series talk?

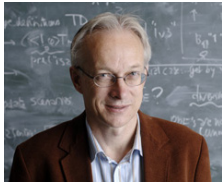
- Just God's-eye-view talk about time and the earlier-later relation $<$.
- To put it another way: talk in classical logic about the flows of time $(T, <)$.
- For example, we can insist that time is irreflexive in this language $\forall t(t \not< t)$.
- As we have already remarked, Prior has already observed that tense logic could not express this property.
- So he was aware of the expressive weakness of his A series language. But at least it was natural. And it least it did not seem to commit him to an ontology of instants, which Prior disliked.

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Hans Kamp showed that Priors system was not as strong as the B-language over the real numbers. And he proved a stunning positive result: the B-language over Dedekind-complete orders could be captured by an A series language by using instead the two place operators **Until** and **Since**. Kamp's operators were destined to become the best known of all modal/temporal operators (computer scientists picked up on them about 15 years later) and also heralded a new dawn in modal logic: interest in **expressivity**.

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Three pillars of modal wisdom, namely: completeness theory, duality theory, and **correspondence theory**.

Doing it in the B language...

Consider the modal representation

$$Fmia - \text{unconscious}$$

we could use instead the first-order representation

$$\exists t(t_0 < t \wedge MIA - \text{UNCONSCIOUS}(t)).$$

This is an example of a systematic translation called the **standard translation**. It build a bridge between modal logic and classical logic. Van Benthem (and many others) have used this bridge to prove many beautiful results.

Standard Translation

And in fact, **any** modal representation can by converted into an equisatisfiable first-order representation:

$$\begin{aligned} ST_t(p) &= Pt \\ ST_t(\neg\varphi) &= \neg ST_t(\varphi) \\ ST_t(\varphi \wedge \psi) &= ST_t(\varphi) \wedge ST_t(\psi) \\ ST_t(F\varphi) &= \exists s(t < s \wedge ST_s(\varphi)) \end{aligned}$$

The language into which we have translated is usually called the (first-order) **correspondence language**.

Prior knew this bridge

Prior also had a correspondence language. Indeed (bar a little notation) it was identical:

Prior wrote $t < s$ as $U(t, s)$

Prior wrote Pt as $T(t, p)$

He called the result the **U-calculus**, or the **UT-calculus**, and it was his basic (indeed, pretty much his only) technical tool.

And certainly by the mid-sixties, probably earlier, he was aware that his tense logic was weaker than his tense logic. And he looked for, and found a solution. That solution was **hybridization** (contemporary terminology, not Prior's).

Basic hybrid logic



Hybrid logic has been independently reinvented several times. A particularly noteworthy example is the Sofia school's work here, conducted by Solomon Passy, Tinko Tinchev, the late George Gargov, and Valentin Goranko.

The basic item on the menu in all these reinventions is the **nominal** (again, contemporary terminology: don't look for it in Prior's writings). A nominal is simply a **syntactically marked** propositional symbol that is true at exactly one time. That is, we have a two-sorted language: some atoms bear 'namelike' information.

We already have a richer logic

Consider the orthodox formula

$$F(r \wedge p) \wedge F(r \wedge q) \rightarrow F(p \wedge q)$$

This is easy to falsify.

We already have a richer logic

Consider the orthodox formula

$$F(r \wedge p) \wedge F(r \wedge q) \rightarrow F(p \wedge q)$$

This is easy to falsify.

On the other hand, the hybrid formula

$$F(i \wedge p) \wedge F(i \wedge q) \rightarrow F(p \wedge q)$$

is **valid** (unfalsifiable). Nominals name, and this adds to the expressive power at our disposal.

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Extension #2

- Add formulas of form $E\varphi$.
- Such formulas assert that φ is at some time.
- Expressions of the form $E(i \wedge \varphi)$ assert that φ holds at the time named i .
- Note: this captures the effect of Prior's $T(i, p)$ — and in a modal language.

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Hybrid Logic in “Past, Present and Future”

“Past, Present and Future”, Arthur Prior, Clarendon Press, 1967.

Chapter 5, Section 6, **Development of the U-calculus within the theory of world states**, pages 88 – 92.

Appendix B, Section 3, **On the range of world-variables and the interpretation of U-calculi in world-calculi**. pages 186 – 197.

Nominals are called world variables, and their interpretations world-propositions. World-calculi means “hybrid logic” and “U-calculi” means “correspondence language”.

Prior is basically interested in showing that hybrid logic can capture the entire correspondence language.

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Hybrid Logic in “Papers on Time and Tense”

“Papers on Time and Tense”, Arthur Prior, Clarendon Press, 1968.

Over a third of the book is about hybrid logic. Four of the five technical papers deal with hybrid logic (the fifth is on metric tense logic). Two of these papers are particularly important for understanding why hybrid logic was important to Prior, and where it led him too.

“The Logic of Ending Time”, pages 98 – 115.

“Tense Logic and Logic of Earlier and Later”, pages 116 – 134.

“Quasi-Propositions and Quasi-Individuals”, pages 135 – 144.

“Tense Logic for Non-Permanent Existents”, pages 145 – 159.

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Hybrid Logic in “Worlds, Times and Selves”

“Worlds, Times and Selves”, by Arthur Prior and Kit Fine, University of Massachusetts Press, 1977.

Posthumously published, essentially a collection of papers and fragments gathered together with an appendix by Kit Fine, “Worlds, Times and Selves” is one of the great might-have-beens of hybrid logic.

This book was to have dealt with “*the interplay between modal and tense logic on the one hand, and quantification theory on the other. One its man concerns was to show that modal and tense logics could stand on their own, that talk of possible worlds or instants was to be reduced these logics rather than conversely.*” From Kit Fine’s Preface to WTS.

Why the neglect of Prior’s work on hybrid logic?

Prior’s use of Polish notation probably didn’t help.

Prior doesn’t carefully demarcate the various logical languages he discusses. They tend to ‘flow’, one into the other. In many ways this is nice — but it may have prevented readers seeing that some of the ingredients introduced along the way were truly novel.

Prior doesn’t really *use* the ideas of hybrid logic for anything apart from solving his philosophical difficulty. Prior doesn’t argue that they are of any independent logical or linguistic interest (an exception can be made for his paper ‘Now’ — which was not in the original edition of *Papers on Time and Tense*). In a nutshell, Prior doesn’t really show that hybrid logic is interesting in its own right — as he so brilliantly did for tense logic.

Talking about time

Reflexivity	$i \rightarrow Fi$	$\forall t(t \prec t)$
Irreflexivity	$i \rightarrow \neg Fi$	$\forall t(t \not\prec t)$
Transitivity	$FFi \rightarrow Fi$	$\forall stu(s \prec t \wedge t \prec u \rightarrow s \prec u)$
Intransitivity	$FFi \rightarrow \neg Fi$	$\forall stu(s \prec t \wedge t \prec u \rightarrow s \not\prec u)$
Symmetry	$i \rightarrow GFi$	$\forall st(s \prec t \rightarrow t \prec s)$
Asymmetry	$i \rightarrow GF\neg i$	$\forall st(s \prec t \rightarrow t \not\prec s)$
Antisymmetry	$i \rightarrow G(Fi \rightarrow i)$	$\forall st((s \prec t \wedge t \prec s) \rightarrow s = t)$
Left-Directedness	PFi	$\forall st\exists u(u \prec s \wedge u \prec t)$
Trichotomy	$Pj \vee i \vee Fj$	$\forall st(s \prec t \vee s = t \vee s \prec t)$
Density	$Fi \rightarrow FFi$	$\forall st(s \prec t \rightarrow \exists u(s \prec u \prec t))$
Discreteness	$i \rightarrow (FT \rightarrow FHH\neg i)$	$\forall st(s \prec t \rightarrow \exists u(s \prec u \wedge \neg \exists v(x \prec v \prec u$

Prior on Reichenbach

Reichenbach offered an influential analysis of tense in natural language based on the idea of **reference points**.

In “Past, Present, and Future”, Prior is rather dismissive of Reichenbach’s approach. He offers no deep criticism. His main point is that sentences like “I shall have been going to see John” require more than one reference point.

Ironically, the hybrid machinery introduced in “Past, Present and Future” allow Prior and Reichenbach’s ideas to be blended seamlessly. Indeed, the hybrid machinery allows what linguists consider the deepest flaw in Reichenbach’s work to be repaired . . .

Prior meets Reichenbach

Structure	Name	English example	Representation
E–R–S	Pluperfect	I had seen	$P(i \wedge P\phi)$
E,R–S	Past	I saw	$P(i \wedge \phi)$
R–E–S	Future-in-the-past	I would see	$P(i \wedge F\phi)$
R–S,E	Future-in-the-past	I would see	$P(i \wedge F\phi)$
R–S–E	Future-in-the-past	I would see	$P(i \wedge F\phi)$
E–S,R	Perfect	I have seen	$P\phi$
S,R,E	Present	I see	ϕ
S,R–E	Prospective	I am going to see	$F\phi$
S–E–R	Future perfect	I will have seen	$F(i \wedge P\phi)$
S,E–R	Future perfect	I will have seen	$F(i \wedge P\phi)$
E–S–R	Future perfect	I will have seen	$F(i \wedge P\phi)$
S–R,E	Future	I will see	$F(i \wedge \phi)$
S–R–E	Future-in-the-future	(Latin: abiturus ero)	$F(i \wedge F\phi)$

It’s not a bug it’s a feature. . .

- As noted, Prior offered a counterexample to Reichenbach’s system.
- Well, it is a counterexample for Reichenbach. . .
- But, ironically, ronically, Prior himself can handle it:
- $F(i \wedge P(j \wedge F(I\text{-see-john})))$.
- Prior’s work actually has the potential to generalize Reichenbach’s.



Recall the Problem: Tense Logic too weak

- As Prior was well aware, tense logic (the A-series language) is **weaker** than the B-series language (the **first-order correspondence language**, or Prior's **UT calculus**).
- This was unacceptable — Prior believed that A-series talk should be able to ground B-series talk. That is, he wanted an A-series language strong enough to swallow all of B-series talk.
- **What to do?**

Strong hybrid logic

- Prior hybridized. He added nominals, the universal modality, and allowed **quantification across nominals**.
- And he gave the **hybrid translation** — **thereby showing that B-series talk was reducible to A-series talk**.
- Let's take a look...

The Hybrid translation

$$\begin{aligned}
 HT(t < s) &= E(t \wedge Fs) \\
 HT(Pt) &= E(t \wedge p) \\
 HT(t = s) &= E(t \wedge s) \\
 HT(\neg\varphi) &= \neg HT(\varphi) \\
 HT(\varphi \wedge \psi) &= HT(\varphi) \wedge HT(\psi) \\
 HT(\exists t\varphi) &= \exists tHT(\varphi) \\
 HT(\forall t\varphi) &= \forall tHT(\varphi)
 \end{aligned}$$

(This translation was known to Prior in the mid 1960s.)

Moreover no more instants...

And hybridization offered more:

... a world-state proposition in the tense-logical sense is simply index of an instant; indeed, I would like to say that it is an instant in the only sense in which 'instants' are not highly fictitious entities (PTT page 188).

- That is, temporal ontology has been banished: **only propositions remain**.
- In many respects, his is an attractive and curiously modern view, with echos of situation theory. It is instructive to read Prior, and consciously substitute the word "information" for "proposition".

Great — so what's the problem?

- As Prior soon realized, however, you could talk about **anything** in this new language'
- For example it could be used to reason about people and their properties and relations. (Prior actually discusses that — arguably he was the first person to do description logic.)
- So not only temporal talk could be reduced to hybrid logic, **any** kind of first order discourse could be. But Prior had wanted to draw a fundamental **distinction** between the two modes of discourse. Suddenly nothing remains of this — **Prior's nightmare**.

Drama!

"Papers on Time and Tense", Arthur Prior, Clarendon Press, 1968.

"Tense Logic and Logic of Earlier and Later", pages 116 – 134. —
Ecstatic: hybrid logic has solved the problem!

"Quasi-Propositions and Quasi-Individuals", pages 135 – 144. —
Houston, we have a problem!

What to do?

In papers of time and tense he explores two options. Here's the first:

Philosophically where do we go from here? We could turn the tables on the objectors to tense logic by saying that only are 'instants' not genuine individuals there no genuine individuals, only certain propositions that can be formally treated as if they they were individuals. [PTT, page 141]

Interesting, and rhetorically attractive ("embrace the nightmare") — but as Prior remarks, most people would probably find this unpalatable. And anyway it doesn't solve his problem as it fails to draw a distinction.

Or this

So far as I can see, there is nothing philosophically disreputable in saying that (i) persons just are genuine individuals, so that their figuring as individuals in a first-order theory needs no explaining (this first-order theory being, on the contrary, the only way of giving sense to its 'modal' counterpart), whereas (ii) instants are not genuine individuals, so that their figuring as values of individual variables does need explaining, and it the related 'modal logic' (tense logic) which gives the first-order logic what sense it has. [PTT, page 142]

In *Papers on Time and Tense* he does not get much farther than outlining the options.

Worlds, Times and Selves (I)

Logicians have tended to welcome the presentation of modal logic as an artificially truncated bit of predicate calculus because we know all about predicate calculus, or at all events know an enormous lot about it, whereas modality is a comparatively obscure and unfamiliar field. And even philosophically, it might be said, it is in general pretty clear what is going on in predicate calculus, but not very clear what is going on in modal logic or even tense logic. [WTS page 56.]

Worlds, Times and Selves (II)

*It is not as simple as this. What we can do with first-order predicate logic in toto is indeed plain enough; but its uniform monadic fragment? Formally, this fragment is no doubt of some interest; for example, unlike the full first-order predicate calculus it is decidable. But what is its philosophical interest? The question, I think partly boils down to this one: What would a philosophically privileged individual be? And to this question, modal and tense logic possibly provide an answer. It is not that modal logic or tense logic is an artificially truncated uniform monadic first-order predicate calculus; the later, rather, is **artificially expanded modal or tense logic**.*

Worlds, Times and Selves (III)

Other interesting answer sketches in WTS — but nothing conclusive.

A great deal of effort devoted to constructing various formal systems, and comparing them — but rather little philosophical discussion in the fragments we have of it.

What can we say from a modern perspective?

- Key difference is the primacy of model theoretic perspective.
- The Amsterdam perspective on modal logic (including hybrid logic) is that we are engaged in an enterprise of exploring fragments of (usually classical) logic from a model theoretic perspective.
- Modal logics are not isolated formal systems. Indeed the goal is to find alternative ways of talking about relational structures, and natural sublogics.
- **But why is this model theoretic perspective interesting?**

Logic returning to its roots

- Antiquity to late 19th century: logic firmly linked to language, knowledge, and cognition. A tool for exploring such issues.
- 20th century. Logic becomes mathematical, and is applied in various branches of mathematics.
- Recent developments: Logics for knowledge representation, logics for natural language semantics, logics for computation ...
- Logic returning to its roots — but stronger than ever. Turned into a genuinely useful tool because of the 20th century mathematical turn.
- **And arguably the model-theoretic perspective is the key.**

Logic the Janus-faced science

- Key insight: to think about language, and representational issues, we need to make two abstractions: **ontology** (models do this for us) and **language** (we have a choice of how to talk about structures).
- Models an appropriate level for thinking about “softer” problems, such as those from natural language semantics and knowledge representations. (We often don’t know much more than that we are dealing with graph-like entities.)
- We then have the chance to explore the variety of ways in which we can talk about, and reason about, such structures — and hopefully we can find ways of doing so that are well behaved mathematically and accord with our intuitions about various problems. **Classic example: “Logic of Time”, by Johan van Benthem.**

So what was Prior's contribution?

- Modern modal logic a valuable tool for the reasons discussed in the first lecture.
- Prior’s insight was that **formulas could be used as terms**. We do **not** need to stick to the traditional logical categories when engaged in applied logic.
- Ultimately, this has showed us how to cut the cake of expressivity along very different lines. There are more interesting options for talking about relational structures than hitherto expected. **We have a larger playground.**

Where should we play?

- **Logic and mathematics?** We've played there a long time (when the mathematicians let us!).
- **Logic and computation?** We're playing there happily already.
- **Logic and language?** A difficult place to play. But, especially since Richard Montague, we're doing quite well.
- **Logic and Philosophy?**

Past, Present and Future...



Happy Birthday ESLLL! See you in 2038!